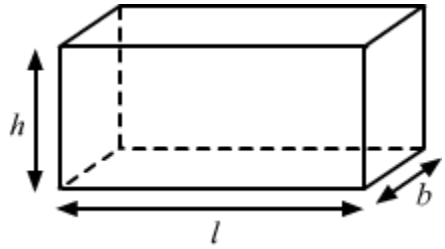
Surface Areas and Volumes

- **Surface areas of cuboid:**



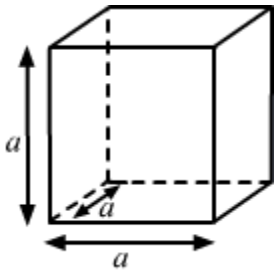
Lateral surface area of the cuboid = $2h\ (l + b)$
Total surface area of the cuboid = $2\ (lb + bh + hl)$

Note: Length of the diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Example: **Find the edge of a cube whose surface area is 294 m².**

Solution: Let the edge of the given cube be a.
$\therefore$ Surface area of the cube = $6a^2$
Given, $6a^2 = 294$
$\Rightarrow a^2 = 49\ \text{m}^2$
$\therefore a = \sqrt{49}\ \text{m} = 7\ \text{m}$

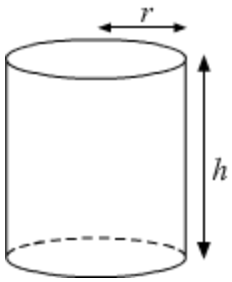
- **Surface areas of cube:**



Lateral surface area of the cube = $4a^2$
Total surface area of the cube = $6a^2$
Note: Length of the diagonal of a cube = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

- **Surface areas of solid cylinder**

- Curved surface area = $2\pi rh$, where r and h are the radius and height
- Total surface area = $2\pi r\ (r + h)$, where r and h are the radius and height



Example : What is the curved surface area of a cylinder of radius 2 cm and height 14 cm?

Solution: Curved surface area of cylinder = $2\pi rh$

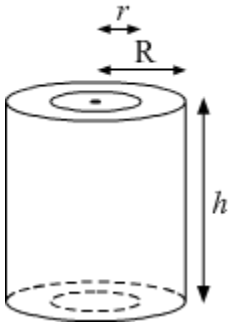
$= 2 \times \frac{22}{7} \times 2 \times 14\,\text{cm}^2$

$= 176\,\text{cm}^2$

- **Surface areas of hollow cylinder**

  - Curved surface area = $2\pi h\ (r + R)$, where r, R and h are the inner radius, outer radius and height
  - Total surface area = CSA of outer cylinder + CSA of inner cylinder + 2 × Area of base

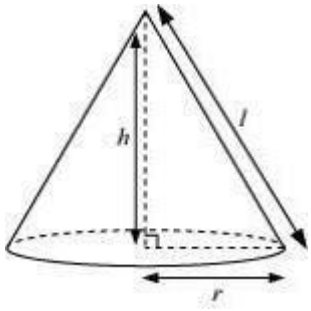
    = $2\pi\ (r + R)\ (\text{h} + \text{R} - r)$, where r, R and h are the inner radius, outer radius and height



- **Surface areas of cone**

  - Curved surface area = πrl, where r and l are the radius and slant height
  - Total surface area = $\pi r\ (l + r)$, where r and l are the radius and slant height

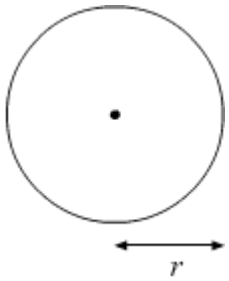
Here, $l = \sqrt{h^2 + r^2}$, where h is the height.



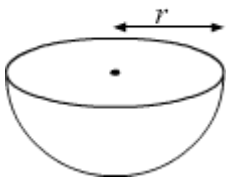
Example: Calculate the curved surface area of a cone of base radius 3 cm and height 4 cm.

Solution: Here, $r = 3$ cm and $h = 4$ cm
$\therefore l = \sqrt{h^2 + r^2} = \sqrt{4^2 + 3^2}\ \text{m} = 5\ \text{cm}$
Curved surface area = $\pi rl = \pi \times 3 \times 5\ \text{cm}^2 = 15\pi\ \text{cm}^2$

- **Surface areas of sphere and hemisphere**
  - Surface area of sphere = $4\pi r^2$, where r is the radius



  - Curved surface area of hemisphere = $2\pi r^2$, where r is the radius
  - Total surface area of hemisphere = $3\pi r^2$, where r is the radius



Example: What is the radius of a balloon whose surface area is 5544 cm^2?

Solution: Let radius of the balloon be r.
Surface area of the balloon = $4\pi r^2 = 5544\ \text{cm}^2$
$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 5544\ \text{cm}^2$
$\Rightarrow r^2 = \frac{5544 \times 7}{88}\ \text{cm}^2$
$\Rightarrow r^2 = 441\ \text{cm}^2$
$\Rightarrow r = \sqrt{441} = 21\ \text{cm}$

Thus, the radius of the balloon is 21 cm.

- **Volume of cube and cuboid**
  - Volume of cube = a^3, where a is the side of the cube
  - Volume of cuboid = $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.

Example: What is the side of a cube of volume 512 cm^3?
Solution: Volume of cube = 512 cm^3

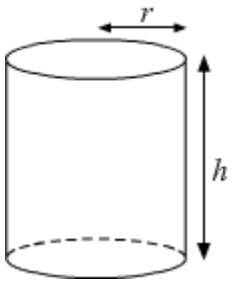
$$\Rightarrow a^3 = 512\,\text{cm}^3$$

$$\Rightarrow a = \sqrt[3]{512}\,\text{cm}^3$$

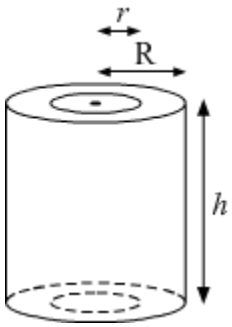
$$\Rightarrow a = 8\,\text{cm}$$

- **Volume of the solid cylinder and hollow cylinder**

  - Volume of solid cylinder = $\pi r^2 h$, where r and h are the radius and height of the solid cylinder



  - Volume of the hollow cylinder = $\pi\,(R^2 - r^2)\,h$, where r, R and h are the inner radius, outer radius and height of hollow cylinder



Example: Find the volume of the pillar of radius 70 cm and height 10 m.

Solution: Radius of the pillar (r) = 70 cm = $\frac{70\text{ m}}{100}$ = 0.7 m

Height of the pillar (h) = 10 m

$$\text{Volume of the pillar} = \pi r^2 h$$
$$= \frac{22}{7} \times (0.7)^2 \times 10\ \text{m}^3$$
$$= 15.4\ \text{m}^3$$

- **Volume of a cone** = $\frac{1}{3}\pi r^2 h$, where r and h are the radius of base and height of the cone.

Example: Calculate the volume of a cone of base radius 3 cm and height 4 cm.

Solution: Here, r = 3 cm and h = 4 cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times 3\ \text{cm} \times 3\ \text{cm} \times 4\ \text{cm}$$
$$= 12\pi\ \text{cm}^3$$

- **Volume of sphere and hemisphere**

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

Example 1: Calculate the volume of a sphere whose surface area is 9π cm^2.

Solution: Surface area = 9π cm^2

$$\Rightarrow 4\pi r^2 = 9\pi$$

$$\Rightarrow r^2 = \frac{9}{4}$$

$$\Rightarrow r = \frac{3}{2}\text{cm}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{3}{2}cm\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right)\text{cm}^3 = 4.5\pi\ \text{cm}^3$$

Example 2: The inner radius of a hemispherical bowl is 4.2 cm. What is the capacity of the bowl?

Solution: Here, r = 4.2 cm

Volume of the bowl $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (4.2cm)^3 = 155.232\ \text{cm}^3$

1 ml = 1 cm^3

Thus, the capacity of the bowl is 155.232 ml.